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Share Ribs and Income Distribution*

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Abstract

The connection between changes in commodity prices and the distribution of income is a question of active interest since the 1941 Stolper—Samuelson Theorem. In higher dimensions results are obtained only if structure is imposed. Here we assume that each of n-industries is alike in the shape of the profile (rib) of distributive factor shares with a permutation of factor numbering such that industry n is most intensive in factor n. Such a structure reveals either a strong version of the Stolper Samuelson Theorem or a Neighborhood oscillation pattern depending on the shape of the share ribs.

1. Introduction

The Stolper-Samuelson Theorem (1941) has exhibited remarkable qualities of endurance, captured, in part, by the conference assembled at Ann Arbor, Michigan in 1991 to celebrate its golden jubilee. The broad question with which they were concerned, viz., the way in which changes in relative commodity prices via the imposition of a tariff, could affect the distribution of income (in particular, the real wage) is one of perennial interest to economists, especially in this day and age in which rent seeking seems to be a ubiquitous phenomenon, and the use of government policies to alter the distribution of income is in widespread evidence. However, a twin set of frustrations has been the reward of economists anxious to generalize the results obtained in the earlier literature. On the one hand, although unambiguous conclusions as to income distribution could be obtained by sticking with the small-scale versions of the Heckscher-Ohlin trade model, the dimensionality of the model seemed too small for useful empirical work or policy prescriptions. On the other hand, enlarging the scale of the model by expanding the number of factors of production and the number of commodities led to the awkward conclusion, common to the pursuit of comparative-statics issues in a general-equilibrium framework, that almost any result could be obtained. As the work of Chipman (1969) and Kemp and Wegge (1969) showed, attempts to generalize the Stolper-Samuelson results even to the case of 4 commodities and factors failed in the face of counterexamples provided by the authors themselves.

Precise links between commodity-price changes and factor-price changes are obtainable in higher dimensions only if strong structure on technology is imposed. If the number of commodities matches the number of productive factors in a set of productive activities exhibiting constant returns to scale, competitive markets, and the lack of joint production (the scenario in which the original investigation was pursued), it is clear from the earlier literature that the key aspect of technology that is crucial involves a comparison of the factor intensities in various sectors. For local results such intensities are efficiently expressed by the matrix of distributive factor shares. The structure which we impose in the present paper is that every sector is

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very much like every other sector in the pattern of distributive shares, subject to a permutation of which factor is most important, which next, and all the way down the line of what may be termed the distributive "share rib" for that sector. More specifically, consider the first industry: suppose the factors are numbered so that factor 1 has the highest share, factor 2 the next highest, and so on to the least intensively used factor, n. Such a weakly monotonic numbering scheme imposes no restrictions on the economy's technology. (We also consider orderings of monotonically increasing shares). The structure we wish to analyze posits that every industry has a distributive *share rib* that has precisely the same shape as that in the first industry, save for a factor permutation such that industry j uses factor j most intensively, factor $(j + 1)$ next, and so on, with factor $(j - 1)$ used least intensively. This means that the θ' matrix of distributive factor shares is a *circulant* matrix, with the first row representing a share rib which, starting in each case from the diagonal element, is identical to every other row. Our objective is to analyze how the relationship between factor rewards and commodity prices depends on the *shape* of this share rib.

The benchmark case is that of a rib showing geometric decay (or growth). Let the ratio of the share of the second most intensively used factor to the first be α, and the ratio of any succeeding factor share to that of the preceding factor also be α. If α is less than unity the rib is monotonically decreasing. We are particularly concerned with share ribs that are systematically "flatter" or "steeper" than this benchmark case. In each industry let the ratio of the second most intensively used factor to the first once again be α, but the ratio of the third to the second, and of every subsequent $\theta_{k+1,k}$ to θ_{kk}, be a constant, β. Such a share rib is flatter than the declining benchmark case if $\alpha < \beta \leq 1$, and steeper if $0 \leq \beta < \alpha$.

If an economy exhibits the degree of regularity represented by common share ribs of this type, strong results emerge which cover the entire range of specific models discussed in the international trade literature. Thus models that lead to strong Stolper-Samuelson conclusions (any commodity-price rise leads to a real increase in that factor intensively associated with the commodity, and a decline in all other factor returns) are to be found if declining share ribs are flatter than the benchmark case. A particularly simple case of flat share ribs is the one in which β is unity, for that represents the common share-rib version of the "produced mobile-factor" structure of Jones and Marjit (1985, 1991). By contrast, steep share ribs lead to the weak Stolper-Samuelson property (Chipman, 1969) whereby each price rise unambiguously raises the real return to the assigned intensive factor, but exhibit as well the oscillating pattern of reward changes associated with the "neighborhood" production structure of Jones and Kierzkowski (1986). The latter model corresponds to a zero value for β. Finally, if the share rib is rising but is flatter than the benchmark case, the pattern of reward changes discussed by Inada (1971) emerges: a price rise leads to a single big loser and many nominal gainers. And, once again, steeper share ribs are associated with oscillating factor rewards so that there is a more balanced distribution of gainers and losers when a commodity price is increased.

Recently, sufficient conditions have been established to ensure strong Stolper-Samuelson results (Jones, Marjit, and Mitra, 1993) as well as weak Stolper-Samuelson results (Mitra and Jones, 1992). These conditions compare the degree of intensity with which the factor uniquely associated with an industry is utilized to an aggregate measure of discrepancy among unintensive factors. Section 3, following section 2's discussion of relatively flat share ribs, shows how the sufficient conditions for the strong Stolper-Samuelson result can be considerably weakened in this case. Section 4 discusses steep share ribs and the neighborhood model, while Section 5 briefly examines the case of rising share ribs. Section 6 follows with a survey of how these various special cases can be illustrated with the geometry of "production triangles," and section 7 provides summary remarks on general properties of the common share-ribs case.

2. Relatively Flat Share Ribs and Strong Stolper-Samuelson Results

The share rib for the first industry consists of the following sequence of values for distributive shares:

$$(1/\Delta)\{1, \alpha, \alpha\beta, \alpha\beta^2, \ldots, \alpha\beta^{n-2}\},$$

where Δ is the sum $(1 + \alpha + \alpha\beta + \cdots + \alpha\beta^{n-2})$. For example, the distributive share of the first factor in the first industry, θ_{11}, is $1/\Delta$, and the third factor's share, θ_{31}, is $\alpha\beta/\Delta$. *Throughout this and the next two sections we assume that α is a positive fraction and that* $0 \leq \beta \leq 1$. That is, the first factor is assumed to have the greatest share in the first industry, with factors numbered such that the shares are nonincreasing after that. Regardless of the technology utilized by an economy, it would always be possible to exhibit a numbering scheme such that the share rib for the first sector is nonincreasing. Here we assume it takes the special form shown above and that the shape of this rib is identical for all sectors once the factor numbering is permuted, so that factor j's distributive share in industry, j, θ_{jj}, is $1/\Delta$, $\theta_{j+1,j}$ is α/Δ, etc. When these share ribs form the rows of a matrix, a particularly simple version emerges of what in the mathematics literature is called a "circulant" matrix.[1]

The key question we wish to raise for economies exhibiting this structure concerns the *shape* of this share rib and how the impact on the distribution of income among factors consequent to a rise in some commodity's price depends upon this shape. In this section, we establish the result that if β lies between α and unity, leading to a relatively flat share rib, the strong version of the Stolper-Samuelson theorem holds. That is, a rise in a commodity's price unambiguously raises the real return to the factor used most intensively in that sector and reduces all other factor rewards. Indeed, we establish a stronger result concerning factor intensities and factor rewards: *There is a perfect positive correlation between changes in factor returns and factor-intensity rankings in the favored sector.*

Two boundary cases deserve special mention. If $\beta = 1$, all the unintensively used factors in any industry exhibit the same distributive share. This structure is a special case of the produced mobile-factor structure discussed by Jones and Marjit (1985, 1991). This latter structure (more general than the $\beta = 1$ case of common share ribs) always yields strong Stolper-Samuelson results. At the other extreme, if $\beta = \alpha$ the boundary result is obtained wherein any commodity-price rise indeed leads to a magnified increase in the return to the most-intensive factor and a fall in the return to the least-intensive factor, but all factors in between find their rewards unaltered.

In what follows we simplify matters by restricting attention to the case in which p_1 rises, all other prices remaining unchanged. Our method of proof utilizes the competitive-profit equations of change whereby the pressures of the competitive marketplace ensure that the relative change in unit costs in any sector matches the relative change in the output price in that sector. Let $\hat{x}$ denote the relative change in a variable, dx/x. Furthermore, let w_i represent the return to factor i. The competitive profit equations of change for sectors k and $k + 1$, where $k = 2, \ldots, n - 2$, are shown by:[2]

$$\alpha\beta^{n-k}\hat{w}_1 + \alpha\beta^{n-k+1}\hat{w}_2 + \cdots + \hat{w}_k + \alpha\hat{w}_{k+1} + \cdots + \alpha\beta^{n-k-1}\hat{w}_n = 0, \quad (1)$$

$$\alpha\beta^{n-k-1}\hat{w}_1 + \alpha\beta^{n-k}\hat{w}_2 + \cdots + \alpha\beta^{n-2}\hat{w}_k^{+}\hat{w}_{k+1} + \cdots + \alpha\beta^{n-k-2}\hat{w}_n = 0. \quad (2)$$

Multiply equation (2) by β and then subtract it from equation (1) to obtain $(1 - \alpha\beta^{n-1})\hat{w}_k + (\alpha - \beta)\hat{w}_{k+1} = 0$; alternatively written,

$$\hat{w}_k = \gamma\hat{w}_{k+1}, \quad \text{where } \gamma \equiv (\beta - \alpha)/(1 - \alpha\beta^{n-1}). \quad (3)$$

This relationship provides the foundation for a recursive backward link between $\hat{w}_n$ and $\hat{w}_k$ for $k = 2, \ldots, n-1$:

$$\hat{w}_k = \gamma^{n-k}\hat{w}_n. \quad (4)$$

More can be said about the linkage among the $\hat{w}_i$ where $i \neq 1$. If β exceeds α and is strictly less than unity, γ is a positive fraction. Therefore each $\hat{w}_k$ is a dampened reflection of $\hat{w}_n$, with absolute values of $\hat{w}_k$ diminishing as k ($\neq 1$) gets smaller. If β equals unity, all the unintensively used factors in the first industry experience the same alteration in factor rewards. If β equals α, γ goes to zero, so that the rise in the price of the first commodity leads to no change in any $\hat{w}_i$ ($i \neq 1, n$). Only the first- and nth-factor rewards change.[3] Finally, note that if $\beta > \alpha$, ensuring that γ is positive, the returns to factors 2 through n must all move in the same direction and, since factor-price changes must average out to commodity-price changes, $\hat{w}_2, \ldots, \hat{w}_n$ have signs opposite to $\hat{w}_1$.

The competitive-profit equation of change in the first industry is

$$\hat{w}_1 + \alpha\hat{w}_2 + \alpha\beta\hat{w}_3 + \cdots + \alpha\beta^{n-2}\hat{w}_n = \Delta\hat{p}_1. \quad (5)$$

Compare this with the corresponding expression for sector 2:

$$\alpha\beta^{n-2}\,\hat{w}_1 + \hat{w}_2 + \alpha\hat{w}_3 + \cdots + \alpha\beta^{n-3}\hat{w}_n = 0. \quad (6)$$

In going from (6) to (5) note that the coefficient of $\hat{w}_1$ has been raised, while the coefficients of all other $\hat{w}_i$ have been reduced. On the right-hand side there has been an increase from zero (the price change in the second sector) to a positive number. This remark suffices to establish that $\hat{w}_1$ must be positive, and all $\hat{w}_i$ ($i \neq 1$) negative (unless $\beta = \alpha$, in which case all are zero except $\hat{w}_n$). The return to factor 1 must therefore rise by a magnified reflection of the rise in p_1; i.e., $\hat{w}_1 > \hat{p}_1$. Thus the strong form of the Stolper-Samuelson theorem is satisfied for all $0 < \alpha < \beta \leqslant 1$, with the borderline case for $\beta = \alpha$.

Figures 1 and 2 summarize these findings. Figure 1 shows the share rib for the first industry, starting with θ_{11} equal to $1/\Delta$, and θ_{21} equal to α/Δ. The dashed share rib there portrayed is for a value of β strictly between α and 1, with the extreme cases shown by the solid curve ($\theta_{i1} = \alpha^{i-1}/\Delta$) and the horizontal line ($\theta_{i1} = \alpha/\Delta$) for $i \geqslant 2$. Figure 2 illustrates the strong Stolper-Samuelson pattern of factor returns when $\alpha < \beta < 1$ as p_1 alone increases. There is a perfect correlation between the intensity of a factor's use in the first sector and the manner in which its return is altered when the price of the first commodity rises.

3. A Comparison with Sufficiency Conditions

A condition on distributive factor shares that is sufficient to guarantee strong Stolper-Samuelson results is provided in Jones, Marjit, and Mitra (1993). Referred to as the strong factor-intensity (SFI) condition, it stipulates not only that the ratio of factor i's distributive share in industry i to any other factor's share (say θ_{ri}) in industry i

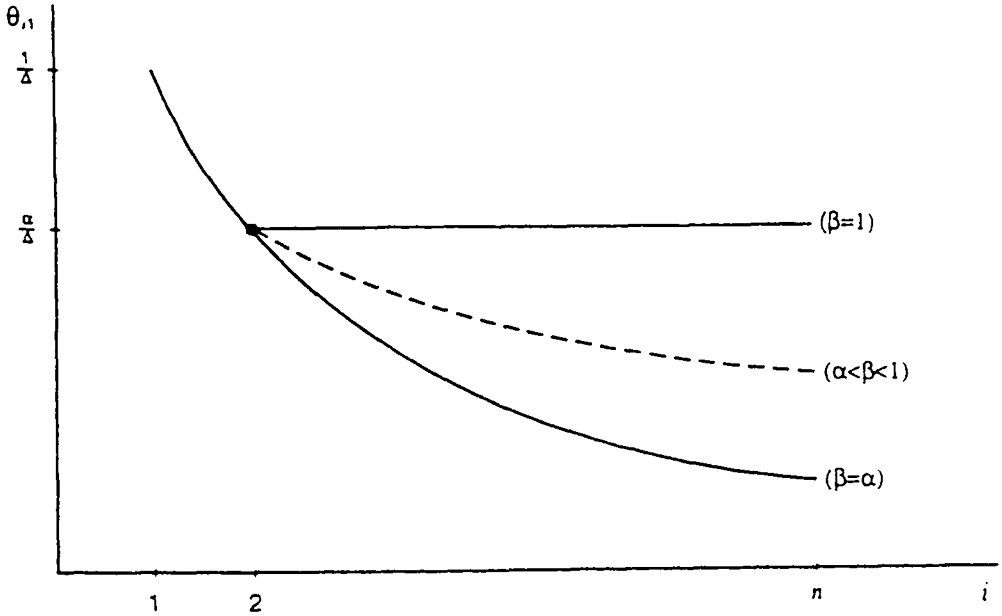
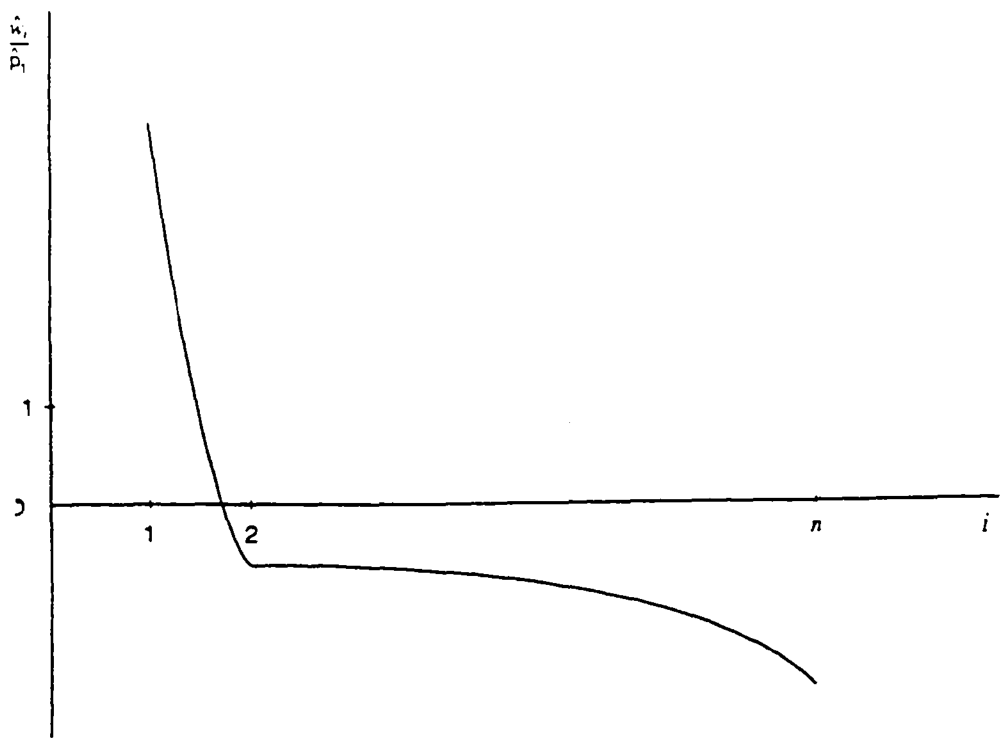


Figure 1. Share Rib for Industry 1



igure 2. Income Distribution: Strong S/S Case

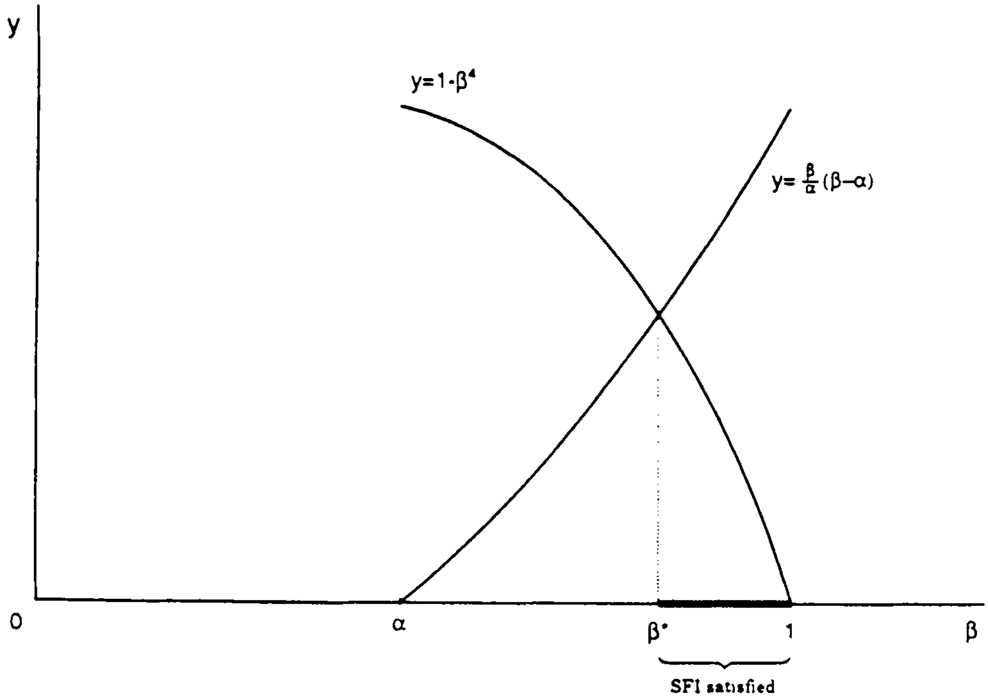


Figure 3. Strong Factor Intensity Condition for n = *4*

exceed the corresponding ratio in any other industry (s), but that this discrepancy be larger than the aggregate absolute value of differences in ratios of other unintensive factors in the two industries. This SFI condition is displayed in (7):

$$\left[\frac{\theta_{ii}}{\theta_{ri}} - \frac{\theta_{is}}{\theta_{rs}}\right] > \sum_{k \neq i,r,s} \left|\frac{\theta_{ki}}{\theta_{ri}} - \frac{\theta_{ks}}{\theta_{rs}}\right|, \quad \text{for } i \neq r, s, r \neq s. \tag{7}$$

We now illustrate how, in the case of falling share ribs (where $0 < \alpha < \beta \leqslant 1$), there is a value of β exceeding α, denoted by β^*, such that this SFI condition is satisfied only if $\alpha < \beta^* < \beta \leqslant 1$, whereas the previous section has shown that the strong Stolper-Samuelson result holds for all values of β between α and unity.

The case of $n = 4$ provides the illustration. Let $i = 1$, $r = 2$, and $s = 3$ in the above formulation of SFI. In the $n = 4$ case there is only one term in the right-hand sum (for $k = 4$), and it equals $(1 - \beta^4)/\beta^2$. The left-hand side of (7) is $(\beta - \alpha)/\beta\alpha$. Therefore the SFI condition for these values of i, r, and s requires $(1 - \beta^4)$ to fall short of $(\beta/\alpha)(\beta - \alpha)$. Figure 3 plots the value of these two expressions in the range $\alpha \leqslant \beta \leqslant 1$ and reveals that the SFI condition is satisfied only if $\beta > \beta^*$. For β in the range (α, β^*) the strong Stolper-Samuelson Theorem is satisfied even though the SFI condition is not. This comparison reveals the consequences of the extra structure provided by the assumption that all sectors have a share rib of the same shape of the form $(1/\Delta)\{1, \alpha, \alpha\beta, \ldots, \alpha\beta^{n-2}\}$, where $0 < \alpha < \beta \leqslant 1$.

4. Steep Share Ribs and the Neighborhood Model

The most sharply declining share rib considered in section 2 corresponds to the case of geometric decay ($\beta = \alpha$), and in such a case a commodity-price rise is accomodated

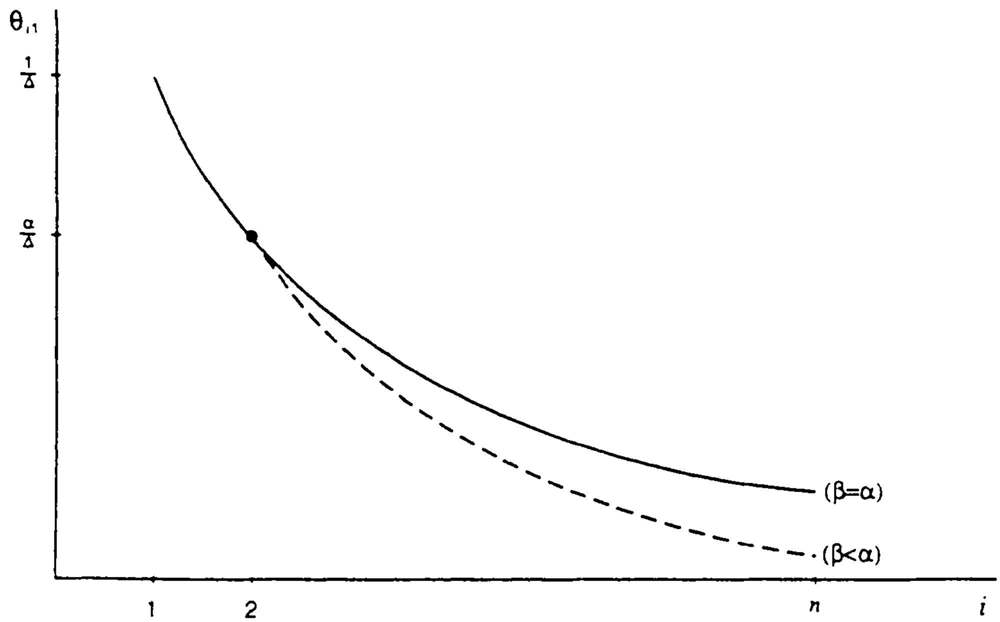


Figure 4. Steeper Share Rib

by a magnified increase in the return to the most intensively used factor and a decline in the return to that factor used least intensively in the favored sector. All other factor returns are unaffected. But steeper share ribs (with $\beta < \alpha$) lead to a rather different income-distribution fallout (See Figure 4). Whereas the strong Stolper-Samuelson results corresponding to flatter share ribs (with $\beta > \alpha$) lead to extreme asymmetry in factor returns as a single price rises (one big winner, with all other factors losing), steeper share ribs lead to a rough balance in the numbers of winners and losers. Once again, any single price rise depresses the return to the factor used least intensively. Thus if p_1 rises, w_n must fall. But now the return to factor $(n - 1)$ rises, although that to $(n - 2)$ falls. The return to the most intensively used factor rises by a magnified amount (what Chipman [1969] termed the weak Stolper-Samuelson result), but the change in the returns to other factors steadily oscillate in sign. This is the "ripple effect" found in the neighborhood model in Jones and Kierzkowski (1986). As we explore below, the latter structure corresponds to the extreme case where $\beta = 0$, although its properties are evident for all $0 \leq \beta < \alpha$.

We start by reaffirming relationship (4), which serves to link all $\hat{w}_k$, $k = 2, \ldots, n - 1$, to the change in factor n's return when p_1 alone rises. The term $\gamma \equiv (\beta - \alpha)/(1 - \alpha\beta^{n-1})$ serves once more as the connecting link between $\hat{w}_k$ and $\hat{w}_{k+1}$, but γ is now negative. It is easy to check that in absolute value γ is still less than unity. The consequence for a rise in p_1 is that $\hat{w}_n$ exceeds in absolute value any other $\hat{w}_k$ $(k \neq 1)$, but working backwards each $\hat{w}_k$ is of opposite sign (although dampened in absolute value) to $\hat{w}_{k+1}$. Such dampened ripples are illustrated in Figure 5.

To probe further, it is necessary to solve explicitly for the returns to extreme factors 1 and n (when p_1 alone rises). This is done by considering the competitive-profit equations of change (i) for commodities 1 and n, and (ii) for commodities 1

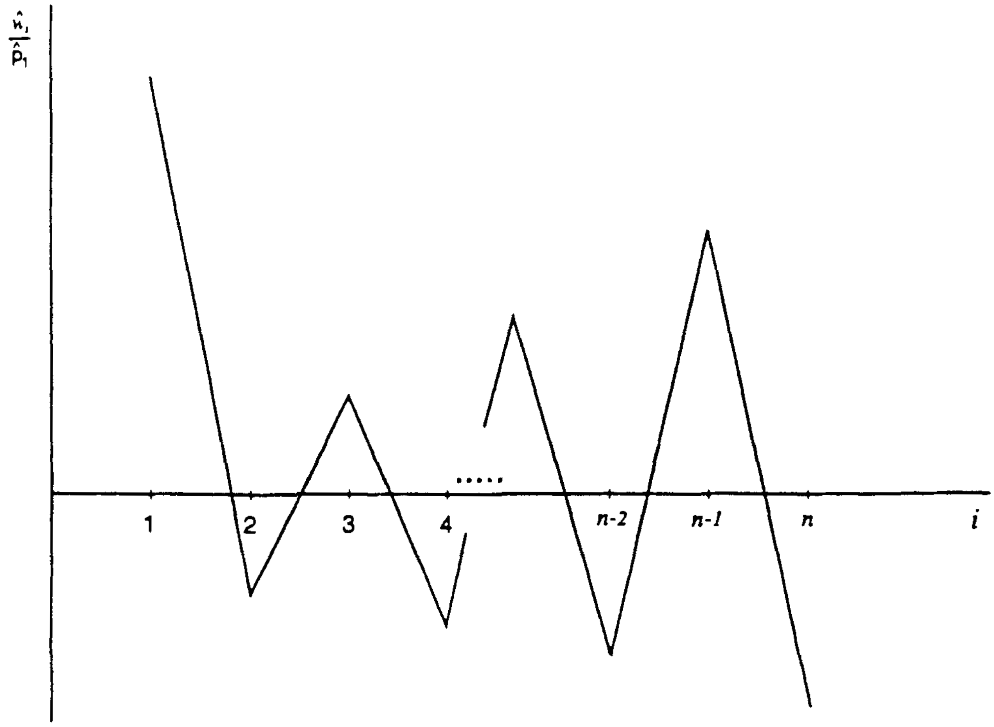


Figure 5. Ripple Effects

and 2, and then using recursive relationship (4) to link $\hat{w}_2$ to $\hat{w}_n$. When p_1 rises, the conditions for commodity n reveal:

$$\alpha\hat{w}_1 + \alpha\beta\hat{w}_2 + \cdots + \hat{w}_n = 0. \tag{8}$$

The conditions for the first commodity have already been displayed in equation (5); multiplying each term by β yields

$$\beta\hat{w}_1 + \alpha\beta\hat{w}_2 + \cdots + \alpha\beta^{n-1}\hat{w}_n = \beta \cdot \Delta \cdot \hat{p}_1. \tag{5'}$$

Subtracting (8) from (5′),

$$(\beta - \alpha)\,\hat{w}_1 - (1 - \alpha\beta^{n-1})\hat{w}_n = \beta \cdot \Delta \cdot \hat{p}_1. \tag{9}$$

The expression Δ refers to the sum of the terms $\{1, \alpha, \alpha\beta, \ldots, \alpha\beta^{n-2}\}$; thus $\beta\Delta$ is the sum of the terms $\{\beta, \alpha\beta, \alpha\beta^2, \ldots, \alpha\beta^{n-1}\}$. Subtracting $\beta\Delta$ from Δ reveals that $(1 - \beta)\Delta = (1 - \alpha\beta^{n-1}) + (\alpha - \beta) = (1 - \alpha\beta^{n-1})(1 - \gamma)$. Solving for Δ and substituting into (9),

$$\gamma\hat{w}_1 - \hat{w}_n = \beta\frac{1 - \gamma}{1 - \beta}\hat{p}_1. \tag{9'}$$

The competitive-profit equations of change for industries 1 and 2 (when p_1 rises) are shown by equations (5) and (6). Multiplying (6) by β and subtracting from (5), $(1 - \alpha\beta^{n-1})\hat{w}_1 + (\alpha - \beta)\hat{w}_2 = \Delta\hat{p}_1$. From (4) it is clear that $\hat{w}_2 = \gamma^{n-2}\hat{w}_n$, so that substituting for $\hat{w}_2$ and Δ, and dividing all terms by $(1 - \alpha\beta^{n-1})$ leads to:

$$\hat{w}_1 - \gamma^{n-1}\hat{w}_n = \frac{1-\gamma}{1-\beta}\hat{p}_1. \tag{10}$$

Equations (9′) and (10) provide the pair necessary to solve for $\hat{w}_1$ and $\hat{w}_n$. The determinant of coefficients is $(1 - \gamma^n)$, which is positive since in absolute value γ always falls short of unity (unless $\beta = 1$). Thus the solution values are

$$\hat{w}_1 = \frac{(1-\gamma)(1-\gamma^{n-1}\beta)}{(1-\gamma^n)(1-\beta)} \cdot \hat{p}_1, \tag{11}$$

$$\hat{w}_n = -\frac{(1-\gamma)(\beta-\gamma)}{(1-\gamma^n)(1-\beta)} \cdot \hat{p}_1. \tag{12}$$

Each of the terms in brackets in (11) and (12) must be positive, establishing that a rise in p_1 must always benefit factor 1 and depress the return to factor n. But as can easily be checked, the return to factor 1 increases by a magnified amount.[4] Figures 4 and 5 illustrate the relationships when n is an even number ($\hat{w}_2$ would be positive if n were odd). Of the positive $\hat{w}_i$, $i \neq 1$, the largest is $\hat{w}_{n-1}$. The appendix proves that when p_1 rises, $\hat{w}_1$ exceeds $\hat{w}_{n-1}$ and so is the largest winner.

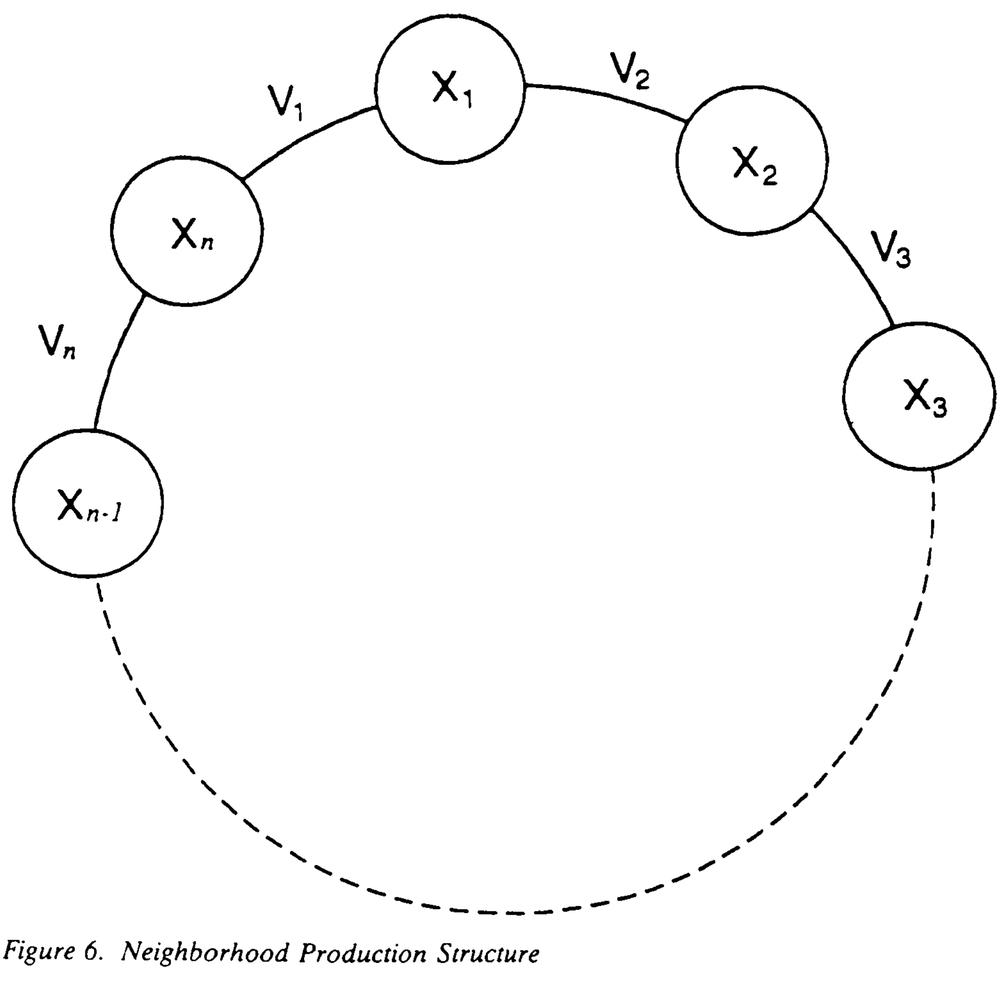


Figure 6. Neighborhood Production Structure

In the neighborhood structure of Jones and Kierzkowski (1986) each productive sector employs only two factors, and each factor has employment opportunities in only two industries. This leads to the general structure illustrated in figure 6, where outputs are denoted by X_j and factors by V_i. Factors and commodities are numbered so that $\Pi\theta_{ii} > \Pi\theta_{i+1,i}$ (where $i + 1$ circles back to unity if $i = n$). This condition suffices to make factor 1 the big winner if the first commodity experiences a price rise in isolation. Since p_n is constant in such a case, a rise in w_1 must lower w_n and, since p_{n-1} is also constant, the fall in w_n must benefit factor $(n - 1)$. These ripples in factor returns are passed back through the chain. Factor 2, the only other factor used in the favored first sector, must lose if n is even, although its reward increases (but by less than p_1 in relative terms) if n is odd. Imposing the regularities of our share-rib structure onto the neighborhood model would entail letting each θ_{ii} equal $1/\Delta$, $\theta_{i+1,i}$ equal α/Δ, with all subsequent factor shares in industry i being zero. That is, the case where $\beta = 0$ is a special case of the neighborhood structure. But the ripple effect of commodity-price changes on factor prices is nonetheless maintained for all $0 \leqslant \beta < \alpha$.

5. Rising Share Ribs: The Inada Case and Factor-Price Oscillations

Suppose share ribs are numbered so that the *least*-intensive factor has its distributive share listed on the diagonal, with monotonically *increasing* values going down the row. Figure 7 illustrates several cases, in all of which $\alpha > 1$. The benchmark case of a geometric rising rib, where $\beta = \alpha$, represents just a renumbering of factors for the benchmark case illustrated in Figure 1. With such renumbering, if the price of the first commodity increases, the return to the first (least intensively used) factor falls, that of the nth factor rises, and all other factor returns remain constant. All other

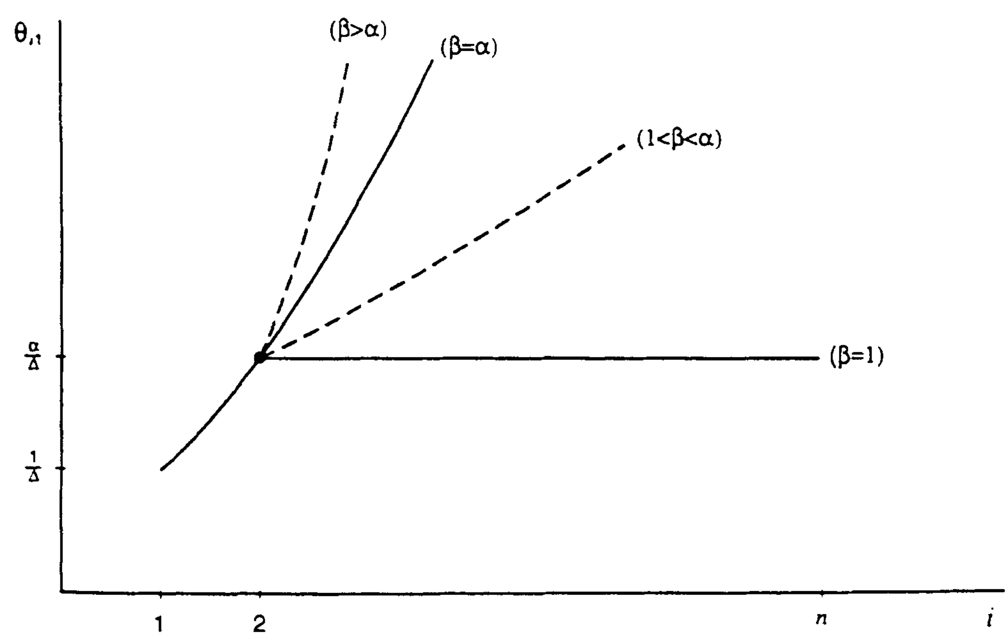


Figure 7. Rising Share Rib

cases represent genuinely different share patterns from those shown for falling share ribs.

In all these cases the returns to factors 2, . . . , n are once again linked in recursive manner by γ, defined precisely as in equation (3). The denominator of γ is now negative, and the benchmark $\beta = \alpha$ case once again divides negative from positive numerators. But an easy check reveals that $|\gamma|$ again falls short of unity (unless $\beta = 1$), so that $\hat{w}_2, \hat{w}_3, \ldots, \hat{w}_{n-1}$ have dampened absolute values relative to $\hat{w}_n$. Equation (12) for $\hat{w}_n$ is again valid (except for $\beta = 1$), but the term $(1 - \beta)$ in the denominator is now negative, ensuring a positive value for $\hat{w}_n$. Also, from (11) it follows that $\hat{w}_1$ is now negative.[5]

Putting this information together, the following behavior patterns emerge:

(i) If $1 < \beta < \alpha$, so that the share rib is flatter than the geometric benchmark case in Figure 7, γ is positive and an increase in p_1 leads to the factor-price changes illustrated in Figure 8. These correspond to Inada's (1971) example whereby every industry is associated with a unique factor whose return is depressed if that industry's price rises, whereas all other factors gain at least in nominal terms. This pattern of one big loser and all others being gainers represents a strong asymmetry and is of just the type opposite to the strong Stolper-Samuelson variety captured by declining share ribs flatter than the benchmark case.

(ii) If $\beta > \alpha$, so that the share rib is steeper than the benchmark case in Figure 7, γ is negative and factor returns exhibit dampened oscillations from the positive value for $\hat{w}_n$. This is similar to the oscillating behavior of the neighborhood type with steep monotonically falling share ribs.

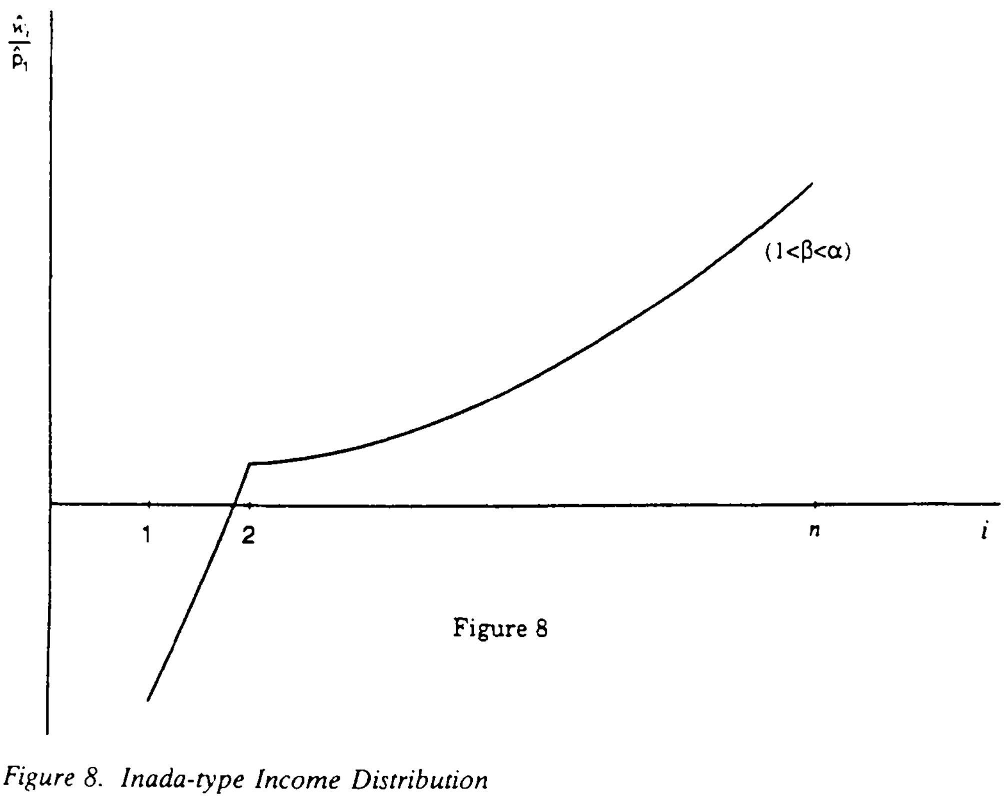
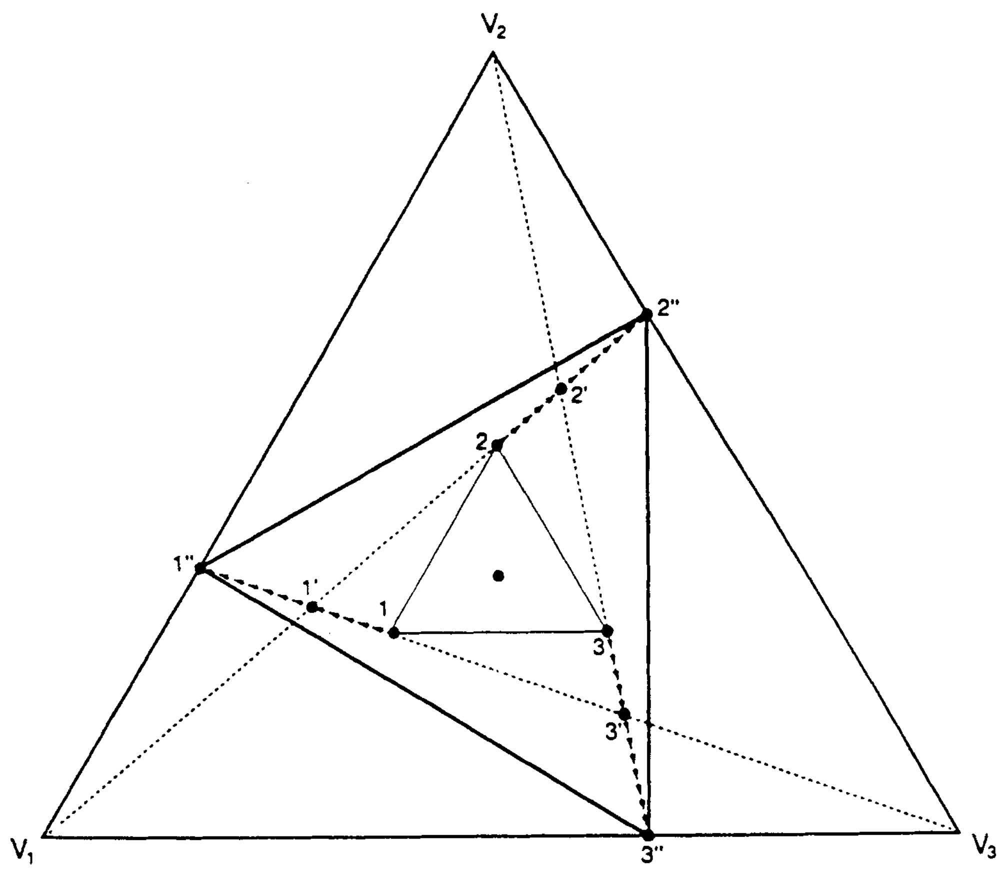


Figure 8. Inada-type Income Distribution

6. Production Triangles: A 3 × 3 Illustration

A useful technique which was pioneered by McKenzie (1955) in discussing factor-price equalization with trade allows a two-dimensional depiction of the three-factor, three-commodity case by using barycentric coordinates with production triangles inscribed in equilateral factor space. This is a technique more recently exploited by Leamer (1987, 1991) and by Jones and Marjit (1991) and Jones (1992) to discuss output responses to factor endowment changes and, through the use of the Samuelson (1953) reciprocity results, to illustrate factor-price responses to commodity-price changes.

The vertices of the factor space in figure 9 correspond to factors V_1, V_2, V_3. Any point in the equilateral triangle represents three magnitudes, shown by the perpendicular distance from the point to each of the three sides. It is a property of equilateral



Δ 123 : $\beta=1$

Δ 1'2'3' : $\beta=\alpha$

Δ 1"2"3": $\beta=0$

$\alpha = \frac{1}{2}$

Figure 9. Production Triangles

triangles that the sum of such perpendicular distances is the same for any point in the triangle and we normalize this to unity. This is a convenient normalization, since it allows any point in the space to represent the distributive factor shares for a productive activity in an economy facing given commodity and factor prices. For example, consider production point 1. The perpendicular distance from this point to the V_2V_3 axis depicts the share of factor V_1 in producing a unit of the first commodity. Similarly, the perpendicular distances from 1 to the V_1V_3 axis and the V_1V_2 axis show, respectively, θ_{21} and θ_{31}. Point 1 has been chosen so that these two shares are equal to each other. Activity points 2 and 3 are symmetrically located so that production triangle 123 is an equilateral triangle whose sides are parallel to the coordinate axes and which is centered in the factor space. With $\theta_{11} > \theta_{21} = \theta_{31}$, β assumes the value of unity, corresponding to the horizontal share rib in Figure 1.

The assumption of this paper, that a common share rib describes activities in all sectors, subject to the appropriate permutation of factor numbering, implies that the production triangle is itself an equilateral triangle. Furthermore, in Figure 9 α has a value of 1/2. This is confirmed by the fact that the distance from point 1 perpendicular to the V_1V_3 axis relative to the distance from point 1 perpendicular to the V_2V_3 axis, θ_{21}/θ_{11}, is 1/2. This ratio is kept constant all along ray V_3 11′1″.

As already noted, β has value unity for the 123 production triangle. This is the symmetric version of the "produced mobile-factor" structure.[6] That the strong version of the Stolper-Samuelson theorem is satisfied is confirmed geometrically in the following fashion. Pick a factor origin, say for factor 2. Draw a ray from this origin through activity point 2 until it hits the side of the production triangle represented by the line 13. (Such a point cuts the line in half). Call this point of intersection A. Then origin V_2, activity point 2, and A are colinear, making point 2 a positive weighted average of V_2 and A. Therefore if techniques and prices remain unchanged, an increase in the economy's endowment of V_2 (alone) could be absorbed completely by an increase in activity 2 and a reduction in (fictitious) activity A. But A is itself a positive convex combination of activities 1 and 3. The upshot: an increase in V_2 would, at unchanged prices, raise x_2 and lower x_1 and x_3. Samuelson's reciprocity result states:

$$\frac{\partial x_j}{\partial V_i} = \frac{\partial w_i}{\partial p_j}.$$

Therefore the alternative exercise of raising any single commodity price must lower the returns to the unintensive factors used in that sector and raise (by a magnified amount) the return to the most intensively used factor.

Lowering the value of β entails a kind of clockwise rotation of the production triangle, enlarging it at the same time to keep the ratio of the share of the second most intensively used factor to the most intensive one equal to α (equal to 1/2). The arrows in Figure 9 show how this is accomplished. For example, activity 1 is moved on ray V_3 11′1″ towards point 1′, activity 2 is moved towards 2′, etc. The variable β reflects, for example, the ratio θ_{31}/θ_{21}, and as 1 moves towards 1′, this ratio is steadily reduced. At 1′ it equals α. The production triangle 1′2′3′ corresponds to the benchmark case in which $\beta = \alpha$. Note that for this triangle, points V_2, 2′ and 3′ are colinear. This implies that an increase in the economy's endowment of V_2 could be absorbed by an increase in 2′, a reduction in 3′, and *no change* is required for the volume of x_1 production. That is, $\partial x_1/\partial V_2$ is zero, and by the reciprocity condition so is $\partial w_2/\partial p_1$, thus confirming for the 3 × 3 case that a price rise for the case of geometrically falling share ribs ($\beta = \alpha$) leaves rewards unchanged for all factors except the most and the least intensively used. (In the 3×3 case there is only one such factor).

Production triangle 1″2″3″ corresponds to the neighborhood production structure in which each sector uses only two factors. For such a triangle an increase in V_1 would, at unchanged techniques, cause x_2 to contract and both x_1 and x_3 to expand. Applying the reciprocity theorem suggests the oscillating pattern of factor rewards when a single commodity price rises.

7. Concluding Remarks

There is no general answer to the question of how factor rewards are indirectly affected by changes in commodity prices. If there is a balance between the numbers of commodity and factor markets, as we have been assuming in our discussion, the pattern of factor intensities, as captured in the matrix of distributive factor shares, is the sole key to the link between these two sets of markets. The structure imposed in this paper is that every productive sector is like any other in terms of the shape of the distributive share rib characterizing factor intensities, with the proviso that each sector has a different factor of production with the highest distributive share, and consequently also has a different ranking of intensities. The focus is on the sensitivity of the commodity-price–factor-price link to the *shape* of the share rib.

One general conclusion emerges: Steep ribs lead to oscillations of factor prices in response to a commodity-price change, whereas flat ribs lead to a redistribution of income whereby a single commodity-price rise leads to one big winner with everyone else losing if the rib is monotonically declining (the strong Stolper-Samuelson property) or one big loser with all other factors gaining at least in nominal terms if the rib is upward sloping (the Inada result). Each of these general conclusions is most readily seen in simple extreme cases. The produced mobile-factor structure in the case of common share ribs is one in which each sector uses a unique factor most intensively and all other, unintensive, factors have the same distributive share—a flat share rib. This imposes a symmetry on the fate of the unintensive factors so that any price rise leads to the intensively used factor gaining at the expense of all the others—a strong Stolper-Samuelson result. At the other extreme lies the neighborhood-production structure, which in the common share-ribs case has each sector using a unique factor intensively, another factor less intensively, and the share rib is so steep that other factors are not used at all. A commodity price rise leads to ripples, oscillations in factor rewards, since a rise in one factor's return in an industry whose price remains constant must lower the return to the other factor employed there, ensuring that, in the next fixed-price sector in which this latter factor is used, the return to the succeeding factor in line is bumped up. The benchmark case which lies between these results is that of a geometrically decaying share rib in which a commodity-price rise can be fully accomodated just by raising the return to the most intensively used factor and lowering the return to the input with the lowest share. All other factor returns are undisturbed.

The structure represented by commonly shaped share ribs allows stronger results to be obtained than are represented by the sufficient conditions in the general case for the strong or weak forms of the Stolper-Samuelson results. Thus in section 3 it was observed that the SFI conditions developed in Jones, Marjit, and Mitra (1993) for the strong form require β to be relatively close to unity, whereas the strong form holds for all $\alpha < \beta \leqslant 1$. Similarly, in Mitra and Jones (1992) sufficient conditions for the weak form of the Stolper-Samuelson result to hold in general cases were derived which in the share-ribs model requires sufficiently high values of β. The results in this paper, however, point to the significantly more robust conclusion that with declining share ribs the return to the most intensively used factor in an industry must rise by a magnified amount for all $0 \leqslant \beta \leqslant 1$, and indeed such a factor must gain relatively more than any other factor. That is, even if factor rewards oscillate in the case of steep share ribs, the most intensively used factor in an industry must have its real reward increased by a commodity-price rise in that sector. In this sense the weak form of the theorem survives intact.

Appendix

We wish to prove that in the case of declining share ribs ($\alpha < 1$), if $\hat{p}_1 > 0$, $\hat{w}_1$ will exceed any other positive $\hat{w}_i$. If $\alpha \leqslant \beta \leqslant 1$, factor 1 is the only gainer and there is nothing left to prove. Therefore suppose $0 \leqslant \beta \leqslant \alpha$. In such a case the absolute value of any $\hat{w}_k$ ($k = 2, \ldots, n-1$) is smaller than any $\hat{w}_{k'}$, for $k' > k$. Since $\hat{w}_n$ is negative, the largest positive $\hat{w}_i$, $i \neq 1$ is $\hat{w}_{n-1}$. From the solutions in the paper for $\hat{w}_1$, $\hat{w}_n$,

$$\hat{w}_1/\hat{p}_1 = (1-\gamma)(1-\gamma^{n-1}\beta)/(1-\gamma^n)(1-\beta),$$

$$\hat{w}_{n-1}/\hat{p}_1 = (1-\gamma)(-\gamma)(\beta-\gamma)/(1-\gamma^n)(1-\beta).$$

Each bracketed expression is positive since γ is negative when $\beta < \alpha$. Therefore to prove that when p_1 rises, $\hat{w}_1 > \hat{w}_{n-1}$, it is only necessary to prove that

$$(1-\gamma^{n-1}\beta) > (-\gamma)(\beta-\gamma).$$

First we note that

$$(1-\gamma^{n-1}\beta) \geqslant \{1-(-\gamma)^{n-1}\beta\}$$

since the equality holds if n is odd, whereas if n is even the left-hand side exceeds unity. Therefore it suffices to establish that

$$\{1-(-\gamma)^{n-1}\beta\} > (-\gamma)(\beta-\gamma),$$

or, dividing by $(-\gamma)$, to prove that

$$1/(-\gamma) > (-\gamma)^{n-2}\beta + \beta + (-\gamma).$$

Next, since $(-\gamma)$ is a fraction, $(-\gamma)$ is at least as large as $(-\gamma)^{n-2}$ for $n > 2$, so that it suffices to prove that

$$1/(-\gamma) > \beta + (-\gamma)(1+\beta).$$

A lower bound can be put on $1/(-\gamma)$, since by definition,

$$1/(-\gamma) \equiv \frac{1-\alpha\beta^{n-1}}{\alpha-\beta},$$

and the minimum value of the right-hand side is reached when α is unity. Thus for $\alpha < 1$,

$$1/(-\gamma) > \frac{1-\beta^{n-1}}{1-\beta}.$$

Note that the latter term is the series sum

$$S \equiv 1 + \beta + \cdots + \beta^{n-2}.$$

With this result at hand it is sufficient to establish that

$$S \geqslant \beta + \frac{1+\beta}{S}, \quad \text{or } S^2 \geqslant \beta S + (1+\beta).$$

But $S^2 \equiv S(1 + \beta + \cdots + \beta^{n-2}) \geqslant S + \beta S = \{1 + \beta + \cdots + \beta^{n-2}\} + \beta S \geqslant 1 + \beta + \beta S$, which is the required inequality. Thus for all $0 \leqslant \beta \leqslant 1$ and $0 < \alpha < 1$, a rise in a single commodity price not only raises by a magnified amount the return to the most intensively used factor (the weak Stolper-Samuelson result), but raises that return by relatively more than any other factor's reward.

A simple example shows that it is not necessarily the case that $\hat{w}_1$ will exceed the absolute value of $(-\hat{w}_n)$ when p_1 rises. Suppose $n = 3$ and

$$\theta' = \begin{bmatrix} 0.4 & 0.38 & 0.22 \\ 0.22 & 0.4 & 0.38 \\ 0.38 & 0.22 & 0.4 \end{bmatrix}.$$

Thus the share rib for the first industry is $(1/\Delta)[1, \alpha, \alpha\beta] = [0.4, 0.38, 0.22]$, corresponding to values $\Delta = 2.5$, $\alpha = 0.95$, and $\beta = 11/19$. The inverse of θ' is:

$$(\theta')^{-1} = \begin{bmatrix} (191/73) & (-259/73) & (141/73) \\ (141/73) & (191/73) & (-259/73) \\ (-259/73) & (141/73) & (191/73) \end{bmatrix}.$$

Therefore if $\hat{p}_1 > 0$, $\hat{p}_2 = \hat{p}_3 = 0$, $[\hat{w}_1/\hat{p}_1, \hat{w}_2/\hat{p}_1, \hat{w}_3/\hat{p}_1] = [(191/73), (141/73), (-259/73)]$, illustrating a case in which $(-\hat{w}_3) > \hat{w}_1$.

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Notes

1. This structure of the share matrix is a special case of the class of matrices considered by Minabe (1967), where for each industry j, $\theta_{jj} > \theta_{j\oplus 1,j} \geqslant \theta_{j\oplus 2,j} \ldots \geqslant \theta_{j\oplus n-1,j}$, where $j \oplus i$ means $j + i$ (mod n). For instance, for industry 2, this means $\theta_{22} > \theta_{32} \geqslant \theta_{42} \geqslant \ldots \gamma \geqslant \theta_{n2} \geqslant \theta_{12}$. Notice that this allows the share rib in a given industry to be more general than the particular form we have chosen; it also allows the shape of the share rib to differ across industries. The advantage of our choice is that we are able to display sharp results on how the nature of the commodity-price–factor-price relationship depends on the *shape* of the share rib.
2. If $k = n - 1$, the competitive-profit equation of change has unity as the coefficient of $\hat{w}_n$.
3. In the mathematics literature, circulant matrices and their inverses have been studied quite extensively. In particular, for a circulant matrix with first row $(1, \alpha, \alpha^2, \ldots, \alpha^{n-1})$, where $0 < \alpha < 1$, it has been noted (Willoughby, 1977), that the first column of the inverse has a positive first element, and a negative nth element, the rest of the elements being zero.
4. The inequality $\hat{w}_1/\hat{p}_1 > 1$ follows since $(1 - \gamma)$ exceeds $(1 - \gamma^n)$ and $(1 - \gamma^{n-1}\beta)$ exceeds $(1 - \beta)$.
5. Note in (11) that the term $(1 - \gamma^{n-1}\beta)$ must be positive despite the assumption that β exceeds unity. Since $|\gamma|$ falls short of unity, it suffices to prove that $\gamma\beta < 1$, or $(\beta^2 - 1) > \alpha\beta[1 - \beta^{n-2}]$. The left-hand side of the inequality is positive and the right-hand side negative for rising share ribs.
6. Production triangles for the general produced mobile-factor structure need not be symmetric. However, rays from each origin passing through the activity point intensive in its use of that factor would all intersect at a common point, which in that structure reflects the factor composition of the produced mobile factor. Each commodity is then produced using some positive combination of the produced mobile factor and a single one of the V_i's. See Jones and Marjit (1991). In Figure 9 these three rays intersect at the center of the triangle.